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HW 5

Math 575B
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1 Part II: Theory.

1. Is there any reason for not accepting the numerical scheme

$$x_n - 3x_{n-1} + 2x_{n-2} = h[f_n + 2f_{n-1} + f_{n-2} - 2f_{n-3}] \quad (1)$$

for the solution of $x' = f(t, x)$? Explain.

Solution. Look at the consistency conditions. Method must satisfy p=1 consistency condition

$$-\sum_{i=0}^{1} ia_i + \sum_{i=-1}^{2} b_i = -(-2) + 1 + 2 + 1 - 2 \quad (2)$$

$$= 4 \neq 1 \quad (3)$$

Therefore, consistency conditions are not met $\implies$ method is not consistent and should not be used.

2. Determine whether the problem $x'' = (57 + \sin t)x$ is or is not stiff.

Solution. Need to check the ratio of the eigenvalues. If the problem is stiff, the ratio of the eigenvalues will be large.

$$A = \begin{pmatrix} 0 & 1 \\ 57 + \sin t & 0 \end{pmatrix} \quad (4)$$

is the fundamental matrix of $x'' = (57 + \sin t)x$. Solve $\det(\lambda I - A)$.

$$\lambda_{1,2} = \pm\sqrt{57 + \sin t} \quad (5)$$

$$\implies |\frac{\lambda_1}{\lambda_2}| = 1 \quad (6)$$

which is not large. Therefore, the problem is not stiff.

2 Part III: Experiment.

1. Solve

$$x'' = e^t + x\cos t - (t+1)x \quad (7)$$

$$x(0) = 1, \quad x(1) = 3 \quad (8)$$

using finite difference methods. Make sure you use the sparse matrix facilities and solvers on matlab. Hand in a convergence and consistency report.

Solution.

Running the code yields the following convergence analysis plot.

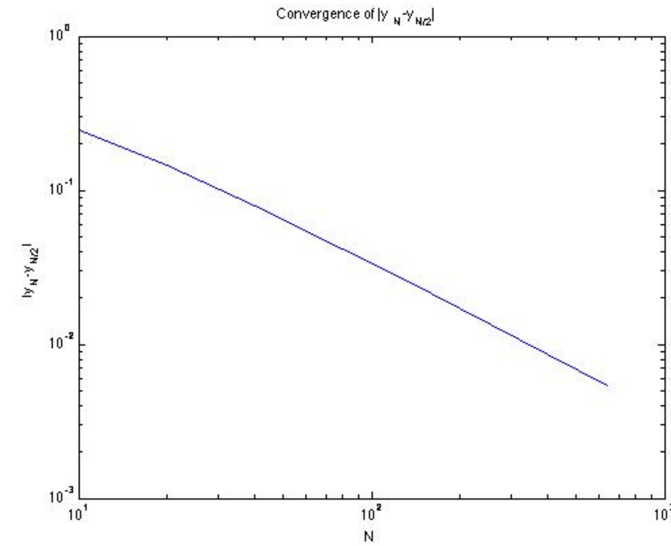

Figure 1: $|y_N - y_{N/2}|$ vs. N. The plot shows convergence to 0 as $N \to \infty$

More than convergence, the algorithm converges quadratically (MATLAB polyfit function yields h^2 convergence). The second order truncation error $\to$ consistency.

2. Solve the beam deflection problem

$$[1 + (y')^2]^{3/2} y'' = \frac{S}{EI} y + \frac{1}{2EI} qx(x - l) \quad (9)$$

$$y(0) = 0 \quad (10)$$

$$y(l) = 0 \quad (11)$$

for $y(x)$, $0 \leq x \leq l$.

Solution.

Running the code yields the following plot.

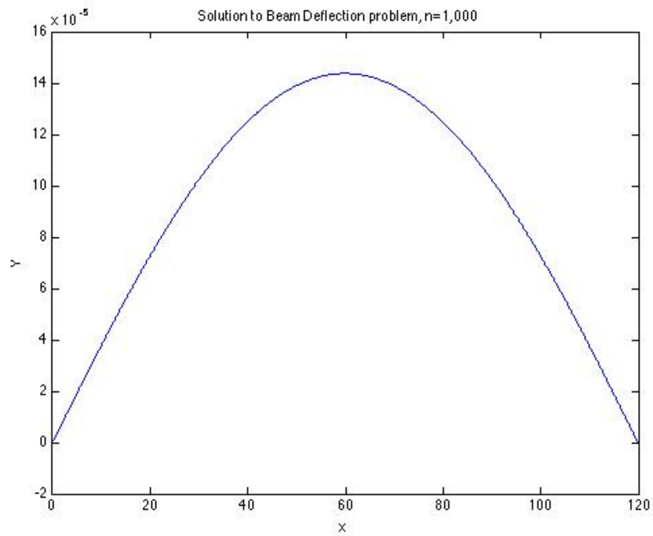

Figure 2: $y(x)$ vs. x for the beam deflection problem

The shooting method was used to make the above plot. RK4 is used as the integrator. Intuitively, the point of greatest deflection should be the midpoint, as indeed it is.